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Improved fixed point method for image restoration

YAN Xue-fei, XU Ting-fa*, BAI Ting-zhu
(*Key Laboratory of Photoelectric Imaging Technology and System of the Ministry of Education, College of Optical Engineering, Beijing Institute of Technology, Beijing* 100081, *China*)
* *Corrponding author, E-mail: xutingfa@*163. *com*

Abstract: We analyze the fixed point method with Tikhonov regularization under the periodic boundary conditions, and propose a changable regularization parameter method. Firstly, we choose a bigger one to restrain the noise in the reconstructed image, and get a convergent result to modify the initial gradient. Secondly, we choose a smaller one to increase the details in the image. Experimental results show that compared with other popular algorithms which solve the L_1 norm regularization function and Total Variation (TV) regularization function, the improved fixed point method performs favorably in solving the problem of the motion degradation and Gaussian degradation.

Key words: image restoration; periodic boundary condition; Tikhonov regularization; change regularization parameter

改进的固定点图像复原算法

阎雪飞,许廷发*,白廷柱
(北京理工大学 光电学院 光电成像技术与系统教育部重点实验室,北京 100081)

摘要:研究了周期边界条件下,Tikhonov 正则化的固定点算法,提出了变化正则化参数的方法。首先对正则化参数取较大值,抑制复原图像中的噪声,通过得出的收敛结果来修正初始梯度;然后对正则化参数取较小值,以增强复原图像中的细节。实验结果表明,与当前求解 L_1 范数正则化函数和全变分正则化函数的流行算法比较,本文算法对于运动模糊与高斯模糊图像的复原效果更佳。

关 键 词:图像复原;周期边界条件;Tikhonov 正则化;变正则化参数
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1 Introduction

In modern society, images are important media for human to obtain the information. However, the images are destroyed inevitably during imaging, copying, and transmission, such as image information is polluted by the noise. While, the high-quality images are necessary in many areas. So the image restoration is a very important task. It is one of the earliest and most classical nonlinear inverse problems in imaging processing, dating back to the 1960′s.

2 Image degradation model

In this class of problems, the image blurring is modeled as:

$$\boldsymbol{g} = \boldsymbol{H}\boldsymbol{f} + \boldsymbol{n} , \tag{1}$$

Where $\boldsymbol{f}(\boldsymbol{f} \in R^{AB\times 1})$ is a $AB \times 1$ vector and represents the unknown $A \times B$ original image. $\boldsymbol{g}(\boldsymbol{g} \in R^{AB\times 1})$ is a $AB \times 1$ vector and represents the $A \times B$ noisy blurred image. $\boldsymbol{n}(\boldsymbol{n} \in R^{AB\times 1})$ is a $AB \times 1$ vector and represents the white Gaussian noise. $\boldsymbol{H}(\boldsymbol{H} \in R^{AB\times AB})$ represents the blur operator. In the case of spatially invariant blurs, it is the matrix representation of the convolution operation. The structure of $\boldsymbol{H}$ depends on the choice of boundary conditions, that is, the underlying assumptions on the image outside the field of view.

We consider the original image:

$$\boldsymbol{f} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} . \tag{2}$$

Zero boundary condition means that all pixels outside the borders are assumed to be zero, which can be pictured as embedding the image $\boldsymbol{f}$ in a large image:

$$\boldsymbol{f}_{\text{ext}} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&2&3&0&0&0 \\ 0&0&0&4&5&6&0&0&0 \\ 0&0&0&7&8&9&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{bmatrix} , \tag{3}$$

In this case, $\boldsymbol{H}$ is a Block Toeplitz with Toeplitz Blocks(BTTB) matrix.

Periodic boundary condition means that the image repeats itself in all directions, which can be pictured as a large image embedded with image $\boldsymbol{f}$:

$$\boldsymbol{f}_{\text{ext}} = \begin{bmatrix} 1&2&3&1&2&3&1&2&3 \\ 4&5&6&4&5&6&4&5&6 \\ 7&8&9&7&8&9&7&8&9 \\ 1&2&3&1&2&3&1&2&3 \\ 4&5&6&4&5&6&4&5&6 \\ 7&8&9&7&8&9&7&8&9 \\ 1&2&3&1&2&3&1&2&3 \\ 4&5&6&4&5&6&4&5&6 \\ 7&8&9&7&8&9&7&8&9 \end{bmatrix} , \tag{4}$$

In this case, $\boldsymbol{H}$ is a Block Circulant with Circulant Blocks(BCCB) matrix.

Reflexive (Neumann) boundary condition means that the scene outside the boundaries is a mirror image of the scene inside the image boundaries, which can be pictured as a large image embedded with image $\boldsymbol{f}$:

$$\boldsymbol{f}_{\text{ext}} = \begin{bmatrix} 9&8&7&7&8&9&9&8&7 \\ 6&5&4&4&5&6&6&5&4 \\ 3&2&1&1&2&3&3&2&1 \\ 3&2&1&1&2&3&3&2&1 \\ 6&5&4&4&5&6&6&5&4 \\ 9&8&7&7&8&9&9&8&7 \\ 9&8&7&7&8&9&9&8&7 \\ 6&5&4&4&5&6&6&5&4 \\ 3&2&1&1&2&3&3&2&1 \end{bmatrix} . \tag{5}$$

In this case, $\boldsymbol{H}$ is the sum of BTTB, Block Toeplitz with Hankel Blocks(BTHB), Block Hankel with Toeplitz Blocks(BHTB) and Block Hankel with Hankel

Blocks(BHHB) matrices[1].

In this paper we propose an improved fixed point method with Tikhonov regularization that we claim performs better than other popular algorithms in both subjective and objective judgements of the image restoration ability. It changes regularization parameters from bigger ones to small ones with iterations, and modifies the initial gradient at every iteration to increase the details in the reconstructed image.

3 Tikhonov regularization function

As we all know, $\boldsymbol{H}$ is often singular or very ill-conditioned. So the problem of estimating $\boldsymbol{f}$ from $\boldsymbol{g}$ is an Ill-posed Linear Inverse Problem(ILIP). A classical approach to ILIP is the Tikhonov regularization which can be expressed as:

$$\boldsymbol{f} = \operatorname{argmin} \frac{1}{2} \| \boldsymbol{Hf} - \boldsymbol{g} \|^2 + \frac{\lambda}{2} \| \boldsymbol{Df} \|^2 . \quad (6)$$

where $\lambda(\lambda > 0)$ is the regularization parameter, $\boldsymbol{D}$ is the penalty matrix. $\| \boldsymbol{Df} \|^2$ is the regularizer or regularization functional, which has many choices such as: L_1 norm regularization functional[2-6], Total-Variation (TV) regularization functional[7-10]. There are algorithms for solving L_1 norm regularization functional such as: Gradient Projection for Space Resonstruction (GPSR) algorithm[11], which includes GPSR-basic algorithm, GPSR-bb algorithm and SpaRSA-monotone algorithm[12]. There are algorithms for solving TV regularization functional such as: TwIST algorithm[13-14], SALSA algorithm[15-16], FTVd algorithm[17-20] and fixed point method[23].

The fixed point method can be expressed as:

$$k = 0 , \quad (7)$$

$$\boldsymbol{f}_0 = 0 . \quad (8)$$

Begin the fixed point iterations:

$$\boldsymbol{D}_k = \boldsymbol{D}(\boldsymbol{f}_k) , \quad (9)$$

$$\boldsymbol{r}_k = \boldsymbol{H}^{\mathrm{T}}(\boldsymbol{Hf}_k - \boldsymbol{g}) + \boldsymbol{\alpha D}_k \boldsymbol{f}_k , \quad (10)$$

$$\boldsymbol{M} = \boldsymbol{H}^{\mathrm{T}}\boldsymbol{H} + \boldsymbol{\alpha D}_k , \quad (11)$$

$$\boldsymbol{s}_k = \boldsymbol{M}^{-1}\boldsymbol{r}_k , \quad (12)$$

$$\boldsymbol{f}_{k+1} = \boldsymbol{f}_k + \boldsymbol{s}_k , \quad (13)$$

$$k = k + 1 . \quad (14)$$

Generally, when $\boldsymbol{r}_k < 0.001$, we stop the iterations.

4 Improved fixed point method

When the boundary condition of image blurring is the periodic boundary condition, we can choose penalty matrix as the negative Laplacian with periodic boundary condition $\boldsymbol{L}$, which is a $B \times B$ partitioned matrix:

$$\boldsymbol{L} = \begin{bmatrix} \boldsymbol{L}_0 & -\boldsymbol{I} & \boldsymbol{\Theta} & \cdots & -\boldsymbol{I} \\ -\boldsymbol{I} & \boldsymbol{L}_0 & -\boldsymbol{I} & \cdots & \boldsymbol{\Theta} \\ \boldsymbol{\Theta} & -\boldsymbol{I} & \boldsymbol{L}_0 & \cdots & \boldsymbol{\Theta} \\ \vdots & \vdots & \vdots & & \vdots \\ -\boldsymbol{I} & \boldsymbol{\Theta} & \boldsymbol{\Theta} & \cdots & \boldsymbol{L}_0 \end{bmatrix} , \quad (15)$$

where $\boldsymbol{I}$ is an $A \times A$ identity matrix, $\boldsymbol{\Theta}$ is a $A \times A$ zero matrix.

$$\boldsymbol{L}_0 = \begin{bmatrix} 4 & -1 & 0 & \cdots & -1 \\ -1 & 4 & -1 & \cdots & 0 \\ 0 & -1 & 4 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -1 & 0 & 0 & \cdots & 4 \end{bmatrix} . \quad (16)$$

We use the matlab function psf = fspecial('motion', 71, 56) to create the motion blurring kernel point spread function, and use g = imfilter(f,psf,'conv','same','circular') to get the motion blurred image; g = imnoise(g,'Gaussian',0, 1×10^{-3}) to add white Gaussian noise of zero mean and standard deviation 1×10^{-3}. We run the fixed point method and compare the reconstructed images when λ is different.

In the Fig. 1, there are 256 pixel × 256 pixel original image Lena(a), motion blurring kernel noisy blurred image(b), and image reconstructed when $\lambda = 10$(c), $\lambda = 1$(d), $\lambda = 10^{-1}$(e), $\lambda = 10^{-2}$(f).

We can find that when we choose a bigger λ, restored image has less noise and also less details; however, when we choose a smaller λ, reconstructed image has more noise and also more details. Therefore, we propose the change regularization pa-

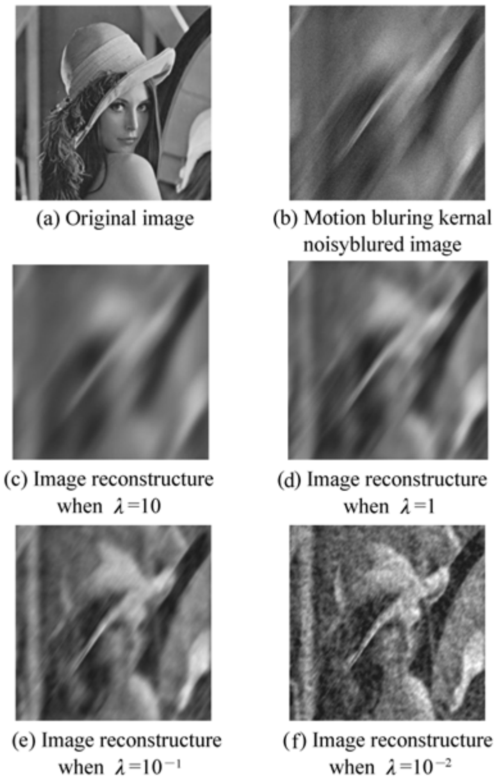

Fig. 1 Image restoration comparison of different regularization parameters

rameter method. Firstly, we choose a bigger λ to restrain the noise in the reconstructed image. Running the fixed point method to get the convergent result $\boldsymbol{f}$, and convolving with $\boldsymbol{H}$ to modify the initial gradient $\boldsymbol{r}_0$ to the real value. We use the new $\boldsymbol{f}$, $\boldsymbol{r}_0$ as the initial ones to run the fixed point method again. Secondly, we choose a smaller λ to increase the details in the reconstructed image. Repeating the above process until the suitable value is found:

1. Give the initial value: $\lambda = \lambda_0$, $k < 1$, $\boldsymbol{f}_0 = 0$, $\boldsymbol{r}_0 = -\boldsymbol{H}^{\mathrm{T}}\boldsymbol{g}$;

2. Run the fixed point method to get the convergent result $\boldsymbol{f}$;

3. $\boldsymbol{f}_0 = \boldsymbol{f}$, $\lambda = \lambda \cdot k$, $\boldsymbol{r}_0 = \boldsymbol{A}\boldsymbol{f}_0 - \boldsymbol{H}^{\mathrm{T}}\boldsymbol{g}$, back to 2.

4 Experimental results

Simulations are carried out to test the proposed algorithm. The Mean Structural Similarity Index Method (MSSIM)[24] is selected to evaluate the restored image quality. The SSIM index is defined as follows:

$$\mathrm{SSIM}(X,Y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)}, \quad (17)$$

Where X and Y are the original and the reconstructed images, respectively; μ_x and μ_y are the mean intensity, σ_x and σ_y are the standard deviation; and σ_{xy} is the covariance.

$$C_1 = (K_1 L)^2, \quad (18)$$

$$C_2 = (K_2 L)^2, \quad (19)$$

Where L is the dynamic range of the pixel values (255 for 8-bit grayscale images), and K_1 and K_2 are small constants.

$$\mathrm{MSSIM}(X,Y) = \frac{1}{M}\sum_{j=1}^{M} \mathrm{SSIM}(x_j, y_j), \quad (20)$$

Where x_j and y_j are the image contents at the j_{th} local window; and M is the number of local windows of the image.

We choose the image Lena as the test target and compare the proposed algorithm with several state-of-the-art algorithms. We choose the GPSR-bb algorithm as GPSR algorithm.

The parameters for the proposed algorithm are $\lambda_0 = 10$, $k = 0.8$. The parameters for the others are defaults in the papers by the authors. The parameters for the GPSR-bb algorithm are $\alpha_{\min} = 10^{-30}$, $\alpha_{\max} = 10^{30}$, $\lambda = 0.35$. The parameters for the SpaRSA-monotone algorithm are $\lambda = 0.035$, $M = 0$, $\sigma = 10^{-5}$, $\eta = 2$. The parameters for the TwIST algorithm are $\alpha = 1.9608$, $\beta = 3.9212$. The parameter for the SALSA algorithm is $\lambda = 0.025$. The parameter for the FTVd algorithm is $\lambda = 50\,000$.

For all the algorithms, we choose the same maximum iteration number as 10000; the same stop criterion as the relative error difference between two iterations is less than 1×10^{-7}. All computations were performed under windows 7 and matlab V 7.10 (R2012a) running on a desktop computer with an Intel Core 530 duo CPU i3 and 2GB of memory.

5.1 Motion degradation

We use the matlab function psf = fspecial('motion', 71, 56) to create the motion blurring kernel point spread function, and use g = imfilter(f, psf, 'conv','same','circular') to get the motion blurred image, and use g = imnoise(g,'Gaussian', 0, 1×10^{-3}) to add white Gaussian noise of zero mean and standard deviation 1×10^{-3}.

In the Fig. 2, there are 256 pixel × 256 pixel original image Lena(a), motion blurring kernel noisy blurred image(b), image restored by the improved fixed point method(c), GPSR-bb(d), SpaRSA-monotone(e), TwIST(f), SALSA(g) and FTVd(h).

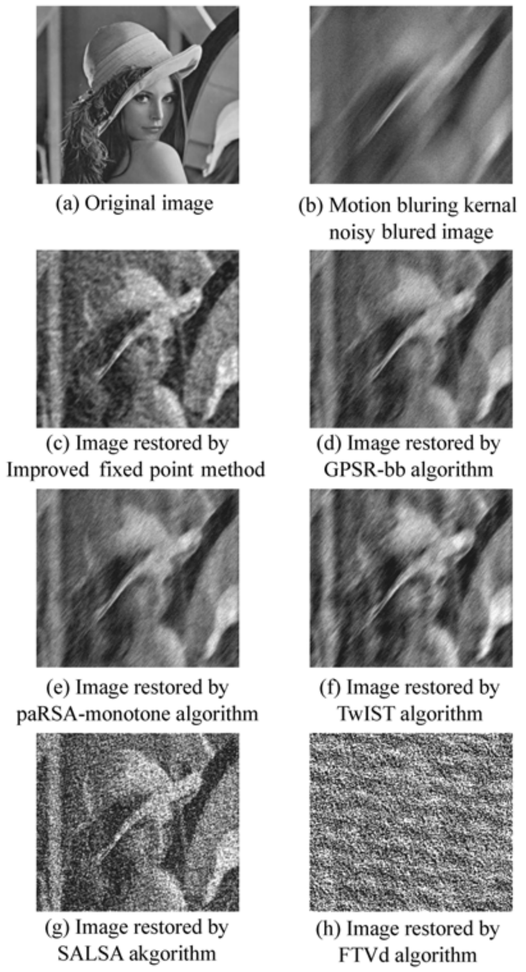

Fig. 2 Restoration comparison of motion degradations

Tab. 1 contains the MSSIM and time comparison between the proposed algorithm and several state-of-the-art algorithms with motion degradation.

Tab. 1 Restoration comparison of motion degradations

Several algorithms	MSSIM	Time/s
Improved fixed point method	0.537 2	4.196
GPSR bb	0.386 9	1.451
SpaRSA-monotone	0.372 1	1.108
TwIST	0.342 7	1.232
SALSA	0.097 3	0.577 2
FTVd	0.000 8	3.931

5.2 Gaussian degradation

We use the matlab function psf = fspecial('Gaussian', [21,28], 10) to create the Gaussian blurring kernel point spread function, and use g = imfilter(f, psf,'conv','same','circular') to get the Gaussian blurred image, and use g = imnoise(g, 'Gaussian', 0, 1×10^{-3}) to add white Gaussian noise of zero mean and standard deviation 1×10^{-3}.

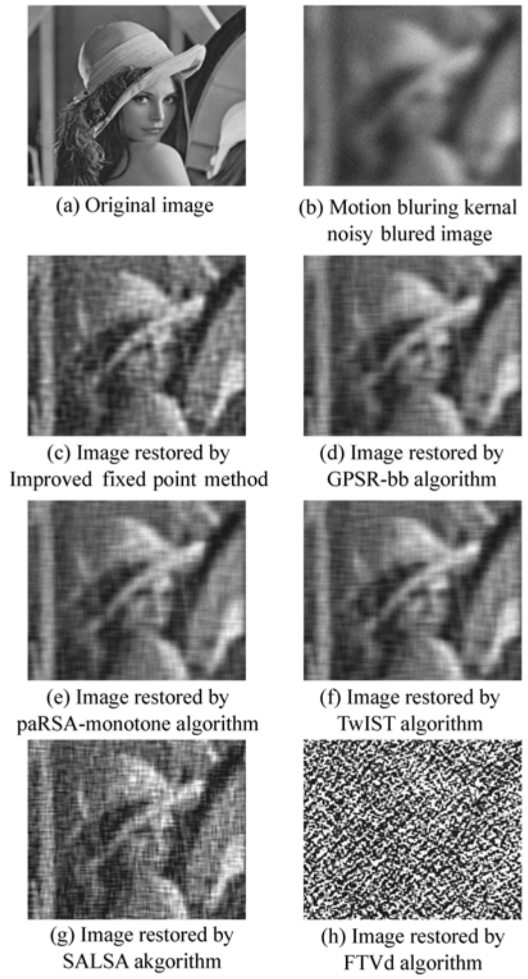

Fig. 3 Restoration comparison of Gaussian degradations

In the Fig. 3, there are 256 pixel × 256 pixel original image Lena (a), Gaussian blurring kernel noisy blurred image (b), image restored by the improved fixed point method (c), GPSR-bb (d), SpaRSA-monotone (e), TwIST (f), SALSA (g) and FTVd (h).

Tab. 2 contains the MSSIM and the time comparison between the proposed algorithm and several state-of-the-art algorithms, with the Gaussian degradation. From the above comparisons, we can see that under the high noise level, the improved fixed point method performs better than the others, and the FTVd algorithm is totally unfit.

Tab. 2 Restoration comparison of Gaussian degradations

Several algorithms	MSSIM	Time/s
Improved fixed point method	0.492 8	4.321
GPSR bb	0.434 4	2.246
SpaRSA-monotone	0.418 7	1.981
TwIST	0.423 9	2.246
SALSA	0.217 5	0.670 8
FTVd	0.001 0	5.476

6 Conclusion

We propose, analyze, and test the improved fixed point method for solving the Tikhonov regularization under the periodic boundary conditions in image restoration problem. In experimental comparisons with state-of-the-art algorithms, the proposed algorithm achieves remarkable performance in both subjective and objective judgments of the image restoration ability.

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Authors' biographies:

YAN Xue-fei (1979—), male, PhD student. He received his B. S. degree in electronics engineering from Shanxi University in 2002, and received his M. S. degree in Optical Engineering from Beijing Institute of Technology in 2007. His research interests include image deblurring. E-mail:yxfamyself@sina.com

BAI Ting-zhu(1955—), male, M. PhD supervisor, Professor, PhD. He received his B. S., M. S. and Ph. D. degrees in Optical Engineering from Beijing Institute of Technology in 1982, 1987 and 2001, respectively. His research interests include electro-optical imaging technology, photoelectric detection technology, and image information processing technology. E-mail:tzhbai@bit.edu.cn

XU Ting-fa(1968—), male, M. PhD supervisor, Professor, post doctorate. He received his B. S. and M. S. degrees in physics from Northeast Normal University in 1992 and 2000, respectively, and received his Ph. D. degree in Optical Engineering from Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences in 2004. He finished studies in mobile postdoctoral station of South China University of Technology in 2006. His research interests include photoelectric imaging detection and recognition, target tracking, photoelectric measurement. E-mail:xutingfa@163.com