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Eigen generalized Jones matrix method

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Abstract: A differential generalized Jones matrix method (dGJM) was recently introduced by Ortega-Quijano and colleagues to derive the GJM for modelling uniaxial and biaxial crystals with arbitrary orientations in laboratory coordinate systems. Later, we propose an eigen generalized Jones matrix method to simulate the phase and polarization of fully polarized light propagating in an anisotropic crystal when the optical axis orientations and light directions are both arbitrary. In our method, we use physics that are equivalent in principle to those of Ortega-Quijano, but we use a modified mathematical technique. We introduce the eigen generalized Jones matrix in the intrinsic coordinate system to precisely calculate the phase and polarization of the light, which overcomes the limitations of the differential generalized Jones matrix method. The simulation results indicate that our method can be used to calculate the polarization distribution, regardless of how the light beam and optical axis positioned, or whether the light beam has a vortex.

Key words: eigen general Jones matrix method; phase and polarization; anisotropic crystal; vortex

本征广义琼斯矩阵方法

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摘要:为了描述完全偏振光在非线性晶体中传播时的偏振态及相位变化,本文基于 Ortega-Quijano 等人在推导非线性晶体的广义琼斯矩阵时采用的微分广义琼斯矩阵方法,提出了本征广义琼斯矩阵方法。与微分广义琼斯矩阵方法相比,本

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征广义琼斯矩阵方法使用了更精确的数学技巧,在描述光在非线性晶体中传播的物理过程上更为严谨。解决了微分广义琼斯矩阵不能计算斜入射光或者光轴与实验室坐标不重合时光的偏振变化的问题。首先,根据折射率椭球方程和光的入射方向,计算出非线性晶体中本征光的传播方向和折射率。然后,给出了本征光的本征广义琼斯矩阵。最后,计算了本征光的偏振态和相位变化。本文使用本征广义琼斯矩阵对带有一个奇点的涡旋光在 KDP 晶体中的传播情况进行模拟计算,计算结果表明,本征广义琼斯矩阵方法能够描述任意入射角度、任意光轴方向的完全偏振光在非线性晶体中的传播过程。

关　键　词:本征广义琼斯矩阵方法;偏振和相位;非线性晶体;涡旋光

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1 Introduction

Jones calculus is a simple and general method for modelling several optical phenomena, such as those of liquid crystal displays[1-2], diffraction gratings[3], Šolc filters[4-6], holographic imaging[7-8], quantum communication [9] in classical and quantum optical fields, radio telescope image calibrators[10], radio polarimeters[11] in astronomical observation, human retinal imaging [12], human brain tissues[13], and biological specimens[14] in the biomedical imaging. Moreover, when applied in three dimensions, the Jones vector changes into the generalized Jones vector[15] and can be used to describe light propagating through a high-numerical-aperture focus lens[16], light interacting with nanoparticles[17], and optical coherence tomography[18].

Jones matrix calculus was first proposed by R. Clark Jones in the 1940s to describe the change in phase and polarization in a matrix or in vector forms for media or light[19]. It is a basic and widely used calculation method for describing the polarization of light transmitting in media. However, it has only been applied to normally or paraxially incident light. Zhang *et al.* introduced a Generalized Jones Vector (GJV), also called a 3D Jones vector to describe the polarization effect of light and optical media or systems[20-24]. Yeh *et al.* extended the method to treat the transmission of off-axis light through an anisotropic medium with an arbitrary optical axis orientation[25]. Azzam *et al.* invented the Generalized Jones Matrix (GJM) to describe the interaction between the fully polarized beam and its linear transformations in three dimensions[26]. Recently, Ortega-Quijano and colleagues proposed the differential Generalized Jones Matrix (dGJM) method to derive the GJM to model uniaxial and biaxial crystals with arbitrary orientations[27-28]. However, our repeated and precise calculations showed that the dGJM method is not applicable to samples with an arbitrary optical axis orientation or when the light is obliquely incident. The reason for this limitation is that the dGJM method tries to get the GJM of an arbitrarily oriented anisotropic crystal in the laboratory coordinate system through the rotation of the GJM consisting of the principle index in principle coordinate system. However, when the light has oblique incidence, the principle index should be replaced by the eigen refraction index, which can be calculated with the n-face equation of the crystal and the direction of the beam in the principle coordinate. Meanwhile, the eigen refraction index can be used to calculate the phase difference of the two eigen polarization lights.

In this paper, we propose a new method for calculating the phase and polarization of fully polarized light propagating in an arbitrarily oriented anisotropic crystal. The method overcomes the limitations of the dGJM method. In Sec. 2, an eigen Generalized Jones Matrix (eGJM) is derived that can be used in uniaxial and biaxial crystals. In Sec. 3, the eGJM is extended to describe the light refraction in the crystal interface. Then, we use the proposed method to simulate the polarization distribution of the cross-section for a light beam with a vortex and compare the results to an image obtained in an experiment[29-30]. The results demonstrate that our method is effective.

2 Eigen generalized Jones matrix method

2.1 Eigen generalized Jones matrix method

To overcome the limitations of the dGJM method, three coordinate systems are necessary: the laboratory coordinate system (S), which describes the position of the crystal; the principal axis coordinate system (Z), which describes the orientation of the optical axis; and the eigen coordinate system (B), which describes the direction of the polarized beam's. In addition, only one eigen coordinate system is required when the light beam transfers in the crystal without any refraction and that only two eigen coordinate systems are required for the two different wave vectors. These coordinates are illustrated in Fig. 1 (color online).

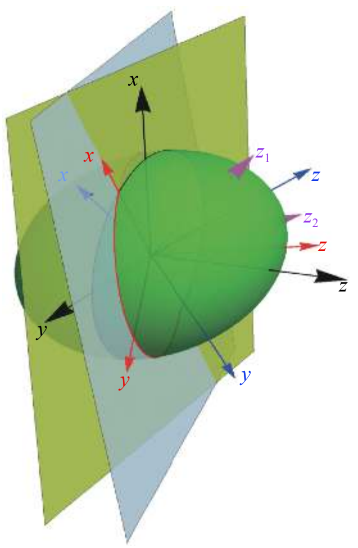

Fig. 1　Schematic diagram of the three coordinate systems. The black, blue, and red axes represent the laboratory, principal, and eigen coordinates, respectively. z_1 and z_2 are the optical axes.

We define the rotation relationship between them as $\boldsymbol{Z}=\boldsymbol{T}_Z\boldsymbol{S}$ and $\boldsymbol{B}=\boldsymbol{T}_B\boldsymbol{S}$, where $\boldsymbol{T}_Z$ and $\boldsymbol{T}_B$ are the rotation matrices, which can be calculated using Euler rotation matrix theory.

To obtain the eigen dGJM, we first calculate the eigen indices n_1 and n_2 from Eq. (1) and Eq. (2) using the principal coordinates:

$$\boldsymbol{K}_i = T_Z\boldsymbol{k}, \tag{1}$$

$$\begin{aligned}&n^4(K_{ix}{}^2+K_{iy}{}^2+K_{iz}{}^2)(K_{ix}{}^2n_x{}^2+K_{iy}{}^2n_y{}^2+K_{iz}{}^2n_z{}^2)\\&-n^2K_{ix}{}^2n_x{}^2(n_y{}^2+n_z{}^2)-n^2K_{iy}{}^2n_y{}^2(n_z{}^2+n_x{}^2)\\&-n^2K_{iz}{}^2n_z{}^2(n_x{}^2+n_y{}^2)+n_x{}^2n_y{}^2n_z{}^2=0,\end{aligned} \tag{2}$$

where n is the refractive index, $\boldsymbol{K}_i$ is the principal wave vector, and n_x, n_y and n_z are the principal indices of the crystal. Eq. (1) is used to rewrite the transporting direction of the eigen light beam in Z, which can directly be used in Eq. (2) and the index face equation in Z.

Second, we can directly write the eGJM in B:

$$\boldsymbol{G}_B=\begin{bmatrix}\exp(-i\delta/2) & 0 & 0\\ 0 & \exp(i\delta/2) & 0\\ 0 & 0 & 0\end{bmatrix}, \tag{3}$$

where $\delta=2\pi(n_1d_1-n_2d_2)/\lambda$ describes the phase difference. d_1 and d_2 are the propagation path lengths of the wave vector for the two eigen lights in the crystal. They must be calculated with different refractions at oblique incidents and identical refrations at normal incidents.

According to the relationship between B and S, the GJM in S is

$$\boldsymbol{G}_S=\boldsymbol{T}_B^{-1}\boldsymbol{G}_B\boldsymbol{T}_B, \tag{4}$$

where $\boldsymbol{T}_B$ is the transfer matrix between the eigen coordinates and laboratory coordinates. The electric displacement vector $\boldsymbol{D}'$ of the output light beam can be expressed as

$$\boldsymbol{D}'=\boldsymbol{G}_S\boldsymbol{D}_i=\boldsymbol{T}_B^{-1}\boldsymbol{G}_B\boldsymbol{T}_B\boldsymbol{D}_i\,. \tag{5}$$

The electric field vector $\boldsymbol{E}'$ can be expressed as

$$\boldsymbol{E}'=(\boldsymbol{T}_Z^{-1}\boldsymbol{\varepsilon}_Z\boldsymbol{T}_Z)^{-1}\boldsymbol{T}_B^{-1}\boldsymbol{G}_B\boldsymbol{T}_B(\boldsymbol{T}_Z^{-1}\boldsymbol{\varepsilon}_Z\boldsymbol{T}_Z)\boldsymbol{E}_i, \tag{6}$$

where $\boldsymbol{T}_Z$ is the transfer matrix between the principal coordinates and laboratory coordinates and $\boldsymbol{\varepsilon}_Z$ is the polarizability tensor in principal coordinates and can be written as

$$\boldsymbol{\varepsilon}_Z = \begin{bmatrix} n_x & & \\ & n_y & \\ & & n_z \end{bmatrix}. \tag{7}$$

The physical meaning of Eq. (6) is easily understood. $\boldsymbol{E}_i$ represents the electric field vector of a light beam in laboratory coordinates and will be transferred to the electric displacement vector $\boldsymbol{D}$ in the same coordinates by the left multiplication of the factor $(\boldsymbol{T}_Z^{-1}\boldsymbol{\varepsilon}_Z\boldsymbol{T}_Z)$. Then, $\boldsymbol{T}_B$ will convert $\boldsymbol{D}$ to eigen coordinates. $\boldsymbol{G}_B$ will change the phase of light, which will finally be reversed to an electric field vector form in laboratory coordinates.

2.2 Uniaxial crystal

We use the eGJM method to calculate the polarization distribution of the light beam in anisotropic crystals.

(1) Beam direction perpendicular to the optical axis

Consider a situation where the direction of the beam is perpendicular to the optical axis. The principal coordinate system is then in the superposition of the eigen coordinate system; thus, $\boldsymbol{T}_B$=$\boldsymbol{T}_Z$. The refractive indices for the eigen beams are exactly the same as the principal index, n_o and n_e. Here, the eGJM is

$$\begin{aligned} \boldsymbol{G}_S &= (\boldsymbol{T}_Z^{-1}\boldsymbol{\varepsilon}_Z\boldsymbol{T}_Z)^{-1}\boldsymbol{T}_{Bu1}^{-1}\boldsymbol{G}_B\boldsymbol{T}_B(\boldsymbol{T}_Z^{-1}\boldsymbol{\varepsilon}_Z T_Z) \\ &= \boldsymbol{T}_Z^{-1}\boldsymbol{\varepsilon}_Z^{-1}\boldsymbol{G}_{Bu1}\boldsymbol{\varepsilon}_Z T_Z, \end{aligned} \tag{8}$$

where

$$\boldsymbol{G}_{Bu1} = \begin{bmatrix} \exp(-i\delta/2) & 0 & 0 \\ 0 & \exp(i\delta/2) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \delta = 2\pi(n_o - n_e)d/\lambda. \tag{9}$$

In this case, there is no walk-off angle between the two eigen beams. Assuming the initial polarization direction is 45° from the x-axis, according to the eGJM method, we can calculate the polarization after a length d. The polarization distribution of the cross-section of the beam is presented in Fig. 2, which shows the change in polarization from the original direction to the opposite polarization direction.

(2) Arbitrary angle between the beam and optical axis

When the angle between the beam and optical axis is arbitrary, the eigen refractive indices for the extraordinary ray will no longer be n_e, but they should be calculated from Eq. (10)[30].

$$n_e(\theta) = \frac{n_o n_e}{[n_o^2 \sin^2\theta + n_e^2\cos^2\theta]^{1/2}}. \tag{10}$$

Then, the eGJM for the electric displacement vector $\boldsymbol{D}$ is

$$\boldsymbol{G}_{SD} = (\boldsymbol{T}_Z^{-1}\boldsymbol{\varepsilon}_Z\boldsymbol{T}_Z)^{-1}(\boldsymbol{T}_B^{-1}\boldsymbol{G}_{Bu2}\boldsymbol{T}_B)(\boldsymbol{T}_Z^{-1}\boldsymbol{\varepsilon}_Z\boldsymbol{T}_Z), \tag{11}$$

where

$$\boldsymbol{G}_{Bu2} = \begin{bmatrix} \exp(-i\delta'/2) & 0 & 0 \\ 0 & \exp(i\delta'/2) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \delta' = 2\pi(n_o - n_{e\theta})d/\lambda. \tag{12}$$

For the extraordinary beam, the array direction is not the same as the wave vector direction, so the eGJM for the electric field vector $\boldsymbol{E}$ should be changed to

$$\boldsymbol{G}_{SE} = (\boldsymbol{T}_Z^{-1}\boldsymbol{\varepsilon}_Z\boldsymbol{T}_Z)^{-1}(\boldsymbol{T}_{Bo}^{-1}\boldsymbol{G}_{Bo}\boldsymbol{T}_{Bo} + \boldsymbol{T}_{Be}^{-1}\boldsymbol{G}_{Be}\boldsymbol{T}_{Be})(\boldsymbol{T}_Z^{-1}\boldsymbol{\varepsilon}_Z\boldsymbol{T}_Z), \tag{13}$$

where

$$\boldsymbol{G}_{Bo} = \begin{bmatrix} \exp(-i\delta_1) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \delta_1 = 2\pi n_o d_1/\lambda, \tag{14}$$

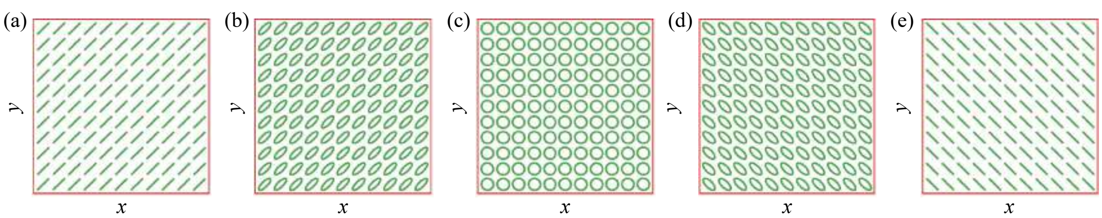

Fig. 2 Spatial distributions of the polarization state. (a) Original linear polarization. (b) Left(right) polarization. (c) Circular polarization. (d) Right(left) polarization. (e) Opposite linear polarization.

$$\boldsymbol{G}_{Be}=\begin{bmatrix}0 & 0 & 0\\ 0 & \exp(-i\delta_2) & 0\\ 0 & 0 & 0\end{bmatrix},\ \delta_2=2\pi n_{e\theta}d_2/\lambda. \quad (15)$$

The walk-off angle should be calculated before we obtain the polarization distribution of the cross-section of the beam. For a potassium dideuterium phosphate (KDDP) crystal, n_o=1.494 2, n_e=1.460 3. The change in walk-off angle with θ_z from 0 to $\pi/2$ is shown in Fig. 3, where the angle between the beam direction and optical axis direction is θ_z.

Figure 3 indicates that the maximum value of the walk-off angle is 0.023 3 rad, equal to 1.335°, and the corresponding refractive index is $n_e(\theta)$= 1.476 96. The walk-off distance is 0.023 3 cm if the beam transfers a distance of 1 cm. Thus, there will be an overlapping region if the size of the beam cross-section is larger than 0.023 3 cm. The polarization distribution for the overlapping region is presented in Fig. 4. The light polarization for the overlapping region could be elliptical, circular, or linear. Meanwhile, the overlapping region decreases in size with the transfer length.

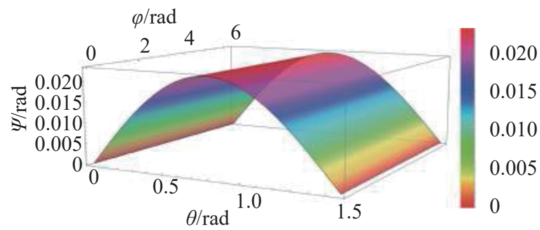

Fig. 3 Change in walk-off angle with θ_z.

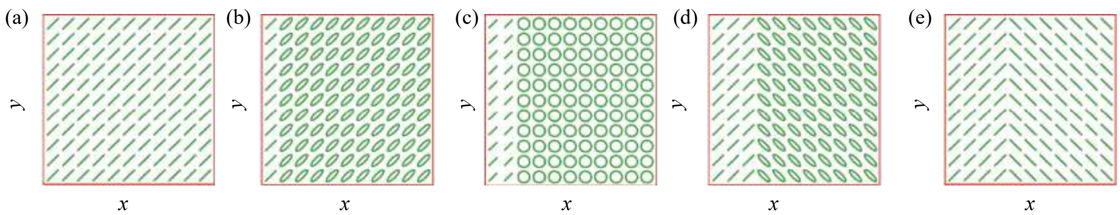

Fig. 4 Spatial distributions of the polarization state with a right direction walk-off effect. (a) Original linear polarization. (b) Left (right) polarization. (c) Circular polarization. (d) Right (left) polarization. (e) Opposite linear polarization.

2.3 Biaxial crystal

In biaxial crystals, there is always a walk-off effect for the light beam so the transmission of light is in the direction of the optical axis and its conical refraction effect is not considered a special situation. The eigen refractive indices for the two eigen linear polarization light beams can be calculated from Eq. (1) and Eq. (2). The eGJM for the electric field vector can immediately be written as

$$\boldsymbol{G}_{SE}=(\boldsymbol{T}_Z^{-1}\boldsymbol{\varepsilon}_Z\boldsymbol{T}_Z)^{-1}(\boldsymbol{T}_{B1}^{-1}\boldsymbol{G}_{B1}\boldsymbol{T}_{B1}+\boldsymbol{T}_{B2}^{-1}\boldsymbol{G}_{B2}\boldsymbol{T}_{B2})(\boldsymbol{T}_Z^{-1}\boldsymbol{\varepsilon}_Z\boldsymbol{T}_Z), \quad (16)$$

where

$$\boldsymbol{G}_{B1}=\begin{bmatrix}\exp(-i\delta_1') & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\ \delta_1'=2\pi n_1 d_1/\lambda, \quad (17)$$

$$\boldsymbol{G}_{B2}=\begin{bmatrix}0 & 0 & 0\\ 0 & \exp(-i\delta_2') & 0\\ 0 & 0 & 0\end{bmatrix},\ \delta_2'=2\pi n_2 d_2/\lambda. \quad (18)$$

The eGJM for the electric displacement vector can be written as

$$\boldsymbol{G}_{SD}=(T_Z^{-1}\boldsymbol{\varepsilon}_Z\boldsymbol{T}_Z)^{-1}(\boldsymbol{T}_B^{-1}\boldsymbol{G}_{Bb}\boldsymbol{T}_B)(\boldsymbol{T}_Z^{-1}\boldsymbol{\varepsilon}_Z\boldsymbol{T}_Z), \quad (19)$$

where

$$\boldsymbol{G}_{Bb}=\begin{bmatrix}\exp(-i\delta''/2) & 0 & 0\\ 0 & \exp(i\delta''/2) & 0\\ 0 & 0 & 0\end{bmatrix},\ \delta''=2\pi(n_1-n_2)d/\lambda. \quad (20)$$

To calculate the polarization distribution of the cross-section of the light beam, we also need to calculate the walk-off angle. We define the light direction as (θ, φ) in principal coordinates, and the change in walk-off angle is shown in Fig. 5 (Color online).

In Fig 5 θ is the polar angle and φ is the azimuth angle. Figures 5(a) and 5(b) are related to the walk-off angle for the light beam with the smaller and larger eigen refractive indices, respectively.

There is no walk-off effect for any eigen light when the light beam transmits in the direction of the axis, corresponding to θ=0, or θ=π/2 and φ is 0, π/2, π, 3π/2. Only one eigen light beam exhibits a walk-off effect when φ is 0, π/2, π, 3π/2 while θ is arbitrary, or when θ=π/2 while φ is arbitrary. There are two singularity points when θ=0.304 and φ is 0, π, corresponding to the optical axis direction. Similar to the uniaxial crystal, the polarization distribution of the cross-section of the light beam is presented in Fig. 6 (Color online). The movement of the different polarizations toward the upper-right quarter represents the array direction.

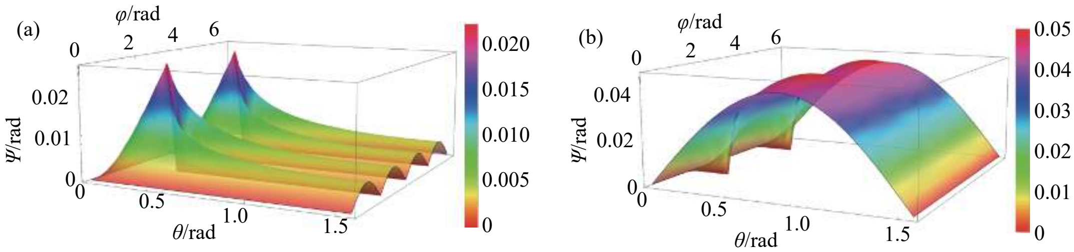

Fig. 5 Change in walk-off angle with (θ, φ)

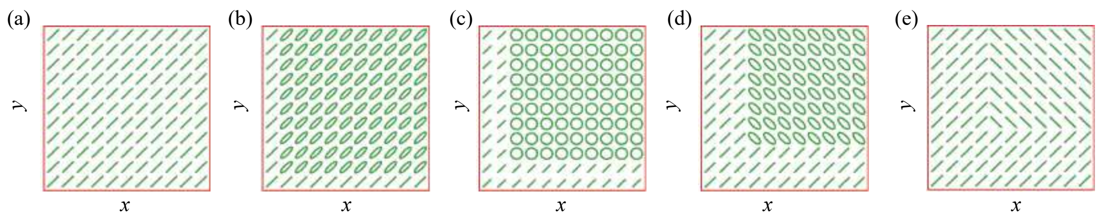

Fig. 6 Spatial distributions of the polarization state with a upward-right direction walk-off effect. (a) Original linear polarization. (b) Left(right) polarization. (c) Circular polarization. (d) Right(left) polarization. (e) Opposite linear polarization.

3 Extended eigen generalized Jones matrix

We extend the eGJM to a more general case of refraction on the interface. Figure 7(a) (Color online) shows the phase difference when the light beam transfers through the anisotropic crystals, where the blue line represents ordinary light, the red line represents the direction of the energy flow of the extraordinary light, and the pink line represents the wave vector direction. We now calculate the phase difference:

$$\begin{aligned}\delta_e-\delta_o &= n_e\cdot d/\cos\theta_e+n\cdot EF\cdot\tan\theta_i-n_o\cdot d/\cos\theta_o\\ &=(n_e\cos\theta_e-n_o\cos\theta_o)d,\end{aligned}\tag{21}$$

$$\begin{aligned}\delta_s-\delta_o &= n_e\cdot d\cos(\theta_e-\theta_s)/\cos\theta_s+n\cdot AF\cdot\tan\theta_i\\ &\quad -n_o\cdot d/\cos\theta_o=(n_e\cos\theta_e-n_o\cos\theta_o)d.\end{aligned}\tag{22}$$

The results indicate that the two phase changes are identical. They are equivalent to either the energy flow direction or the wave vector direction of the extraordinary light. Figure 7(b) shows the different polarization directions at the interface. For the anisotropic crystal, the birefringence must be considered. Yeh[24] already provided a method to calculate the polarization of the output light beam. Assuming the thickness of the crystal is d, we know the optical distance difference is ($n_e\cos\theta_e-n_o\cos\theta_o$) from Eq. (21) and Eq. (22); thus, the output light beam can be expressed as

$$\begin{aligned}A'_s &=(t_{os}T_o\mathrm{e}^{-\mathrm{i}\delta_1/2}+t_{es}T_e\mathrm{e}^{\mathrm{i}\delta_1/2})\mathrm{e}^{-\mathrm{i}\delta_2/2}\\ A'_p &=(t_{ps}T_o\mathrm{e}^{-\mathrm{i}\delta_1/2}+t_{ps}T_e\mathrm{e}^{\mathrm{i}\delta_1/2})\mathrm{e}^{-\mathrm{i}\delta_2/2},\end{aligned}\tag{23}$$

where δ_1=($n_e\cos\theta_e-n_o\cos\theta_o$)d$\omega/c$ and δ_2=($n_e/\cos\theta_e+n_o/\cos\theta_o$)d$\omega/c$. Ignoring the phase factor $\mathrm{e}^{-\mathrm{i}\delta_2/2}$, we have the following matrix form:

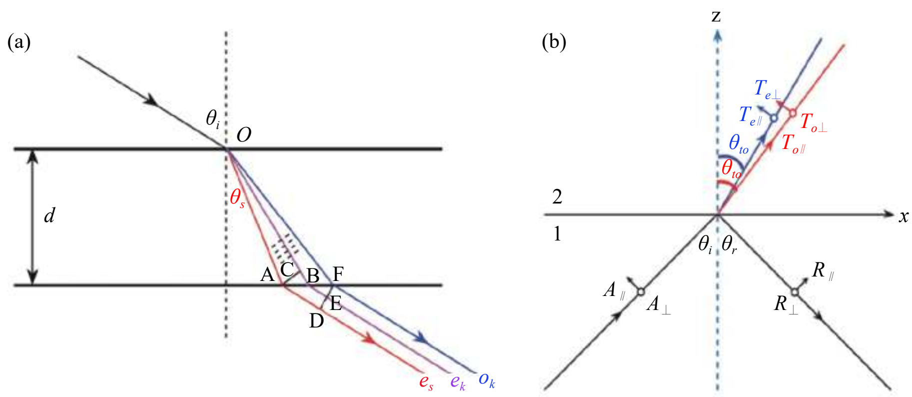

Fig. 7 Phase difference and polarization. (a) Phase difference of the refracted light beam in birefringent crystals. (b) Polarization of reflection and refracted light beam at the interface in birefringent crystals.

$$\begin{pmatrix}A'_s\\A'_p\end{pmatrix}=\begin{pmatrix}t_{os}&t_{es}\\t_{op}&t_{ep}\end{pmatrix}\begin{pmatrix}e^{-i\delta_1/2}&0\\0&e^{i\delta_1/2}\end{pmatrix}\begin{pmatrix}t_{so}&t_{po}\\t_{se}&t_{pe}\end{pmatrix}\begin{pmatrix}A_s\\A_p\end{pmatrix}. \tag{24}$$

If we extend Eq. (24) to three dimensions, we have

$$\begin{pmatrix}E'_x\\E'_y\\E'_z\end{pmatrix}=\boldsymbol{T}\begin{pmatrix}A'_s\\A'_p\\0\end{pmatrix}=\boldsymbol{T}\begin{pmatrix}t_{os}&t_{es}&0\\t_{op}&t_{ep}&0\\0&0&0\end{pmatrix}\cdot\begin{pmatrix}e^{-i\delta_1/2}&0&0\\0&e^{i\delta_1/2}&0\\0&0&0\end{pmatrix}\begin{pmatrix}t_{so}&t_{po}&0\\t_{se}&t_{pe}&0\\0&0&0\end{pmatrix}\boldsymbol{T}^{-1}\boldsymbol{T}\begin{pmatrix}A_s\\A_p\\0\end{pmatrix}=\boldsymbol{G}_S\begin{pmatrix}E_x\\E_y\\E_z\end{pmatrix}. \tag{25}$$

In applications, we calculate the phase distribution for a vector vortex light beam with a singularity transference through the KDP crystal and compare the simulation results to the experimental results of Flossmann[28], as shown in Fig. 8 (Color online).

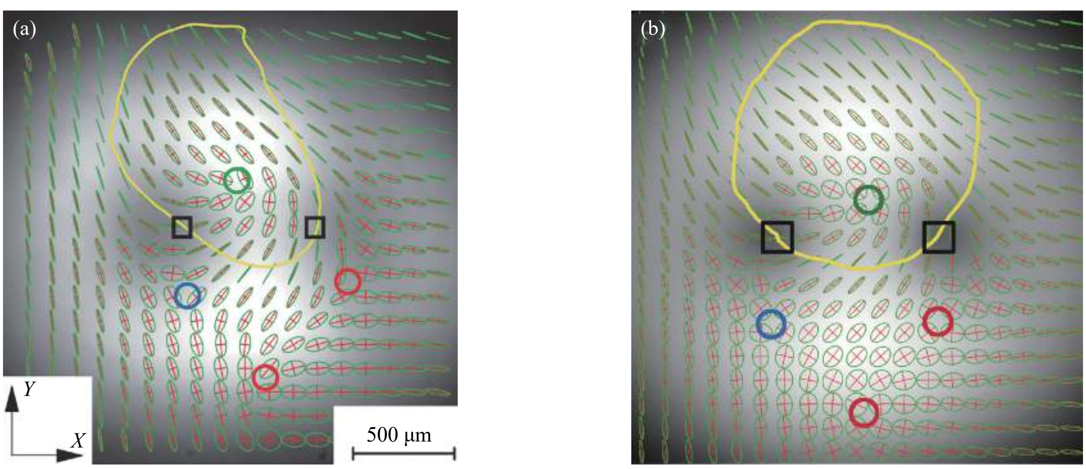

Fig. 8 (a) Experimental image and (b) simulation results of proposed method.

The black squares indicate the singularities of the light beam. The colored circles represent the circular polarization state points and the yellow lines represent the linear polarization states in the cross-section of the output vector beam. There is a small difference in the bottom and middle areas between these two images because of experimental error and simulation method. However, the polarization distribution and positions of the special points are almost identical, which clearly indicates that the eGJM method is practical.

4 Conclusions

In this study, we analyzed the GJM method, which provides a convenient way to establish the

Jones matrix for anisotropic crystals whose optical axis is oriented arbitrarily in three-dimensional space. We proposed the eGJM method to overcome the limitation of the dGJM, which is effective only when the light has perpendicular incidence and the optical axis is perpendicular or parallel to the incidence face. The calculation results indicate that our method can be used to construct the Jones matrix when the directions of the light beam and optical axis are both arbitrary. The eGJM can also be extended to include cases where the light refraction is on the interface when light travels through the crystal, so that its polarization and phase can be precisely calculated. Finally, we use this method to simulate the polarization distribution of the cross-section for a fully polarized light beam with a vortex transmitting through an anisotropic crystal, and we compare the results to those of an experiment. The results demonstrate that our method is effective. Thus, the eGJM method has potential applications in simulating the space evolution of vector beams. Optional optical crystal instruments can be calculated based on the requirement beams. Factors like the electro-photon effect, magnetic-photon effect and optical rotation should be further studied to fully develop the eGJM method for applications like light propagation in crystals in electromagnetic fields.

参考文献:

[1] YU F H, KWOK H S. Comparison of extended Jones matrices for twisted nematic liquid-crystal displays at oblique angles of incidence[J]. *Journal of the Optical Society of America A*, 1999, 16(11): 2772-2780.

[2] LIEN A, CHEN C J. A new 2× 2 matrix representation for twisted nematic liquid crystal displays at oblique incidence[J]. *Japanese Journal of Applied Physics*, 1996, 35(9B): L1200-L1203.

[3] AZZAM R M A, BASHARA N M. Generalized ellipsometry for surfaces with directional preference: application to diffraction gratings[J]. *Journal of the Optical Society of America*, 1972, 62(12): 1521-1523.

[4] CHEN X F, SHI J H, CHEN Y P, *et al*.. Electro-optic Solc-type wavelength filter in periodically poled lithium niobate[J]. *Optics Letters*, 2003, 28(21): 2115-2117.

[5] CHEN L J, SHI J H, CHEN X F, *et al*.. Photovoltaic effect in a periodically poled lithium niobate Solc-type wavelength filter[J]. *Applied Physics Letters*, 2006, 88(12): 121118.

[6] SHI J H, WANG J H, CHEN L J, *et al*.. Tunable Šolc-type filter in periodically poled LiNbO3 by UV-light illumination[J]. *Optics Express*, 2006, 14(13): 6279-6284.

[7] LIU X, YANG Y, HAN L, *et al*.. Fiber-based lensless polarization holography for measuring Jones matrix parameters of polarization-sensitive materials[J]. *Optics Express*, 2017, 25(7): 7288-7299.

[8] PURTSELADZE A L, TARASASHVILI V I, SHAVERDOVA V G, *et al*.. Polarization memory of denisyuk holograms formed in unpolarized light[J]. *Journal of Applied Spectroscopy*, 2014, 81(1): 63-68.

[9] 杨海马, 马彩文, 王建宇, 等. 量子光通信中偏振光空间姿态传递[J]. 光子学报, 2015, 44(12): 1227002.
YANG H M, MA C W, WANG J Y, *et al*.. The transmission of polarized light of space attitude in quantum communication[J]. *Acta Photonica Sinica*, 2015, 44(12): 1227002. (in Chinese)

[10] CAROZZI T D. Simple estimation of all-sky, direction-dependent Jones matrix of primary beams of radio interferometers[J]. *Astronomy and Computing*, 2016, 16: 185-188.

[11] CAROZZI T D, WOAN G. A fundamental figure of merit for radio polarimeters[J]. *IEEE Transactions on Antennas and Propagation*, 2011, 59(6): 2058-2065.

[12] BRAAF B. Fiber-based Jones-matrix polarization-sensitive OCT of the human retina[J]. *Investigative Ophthalmology & Visual Science*, 2016, 57(12).

[13] MENZEL M, MICHIELSEN K, DE RAEDT H, *et al*.. A Jones matrix formalism for simulating three-dimensional polarized light imaging of brain tissue[J]. *Journal of the Royal Society Interface*, 2015, 12(111): 20150734.

[14] YANG T D, PARK K, KANG Y G, *et al*.. Single-shot digital holographic microscopy for quantifying a spatially-

resolved Jones matrix of biological specimens[J]. *Optics Express*, 2016, 24(25): 29302-29311.

[15] SHEPPARD C J R. Jones and Stokes parameters for polarization in three dimensions[J]. *Physical Review A*, 2014, 90(2): 023809.

[16] KANG H, JIA B H, GU M. Polarization characterization in the focal volume of high numerical aperture objectives[J]. *Optics Express*, 2010, 18(10): 10813-10821.

[17] ORLOV S, PESCHEL U, BAUER T, *et al*.. Analytical expansion of highly focused vector beams into vector spherical harmonics and its application to Mie scattering[J]. *Physical Review A*, 2012, 85(6): 063825.

[18] LI E, MAKITA S, HONG Y J, *et al*.. Three-dimensional multi-contrast imaging of in vivo human skin by Jones matrix optical coherence tomography[J]. *Biomedical Optics Express*, 2017, 8(3): 1290-1305.

[19] JONES R C. A new calculus for the treatment of optical systems. VII. Properties of the N-matrices[J]. *Journal of the Optical Society of America*, 1948, 38(8): 671-685.

[20] HE W J, FU Y G, LIU Z Y, *et al*.. Three-dimensional polarization aberration functions in optical system based on three-dimensional polarization ray-tracing calculus[J]. *Optics Communications*, 2017, 387: 128-134.

[21] HE W J, FU Y G, ZHENG Y, *et al*.. Polarization properties of a corner-cube retroreflector with three-dimensional polarization ray-tracing calculus[J]. *Applied Optics*, 2013, 52(19): 4527-4535.

[22] LI Y H, FU Y G, LIU Z Y, *et al*.. Three-dimensional polarization algebra for all polarization sensitive optical systems[J]. *Optics Express*, 2018, 26(11): 14109-14122.

[23] ZHANG H Y, LI Y, YAN CH X, *et al*.. Three-dimensional polarization ray tracing calculus for partially polarized light[J]. *Optics Express*, 2017, 25(22): 26973-26986.

[24] YEH P. Extended Jones matrix method[J]. *Journal of the Optical Society of America*, 1982, 72(4): 507-513.

[25] AZZAM R M A. Three-dimensional polarization states of monochromatic light fields[J]. *Journal of the Optical Society of America A*, 2011, 28(11): 2279-2283.

[26] ORTEGA-QUIJANO N, ARCE-DIEGO J L. Generalized Jones matrices for anisotropic media[J]. *Optics Express*, 2013, 21(6): 6895-6900.

[27] ORTEGA-QUIJANO N, FADE J, ALOUINI M. Generalized Jones matrix method for homogeneous biaxial samples[J]. *Optics Express*, 2015, 23(16): 20428-20438.

[28] FLOSSMANN F, SCHWARZ U T, MAIER M, *et al*.. Polarization singularities from unfolding an optical vortex through a birefringent crystal[J]. *Physical Review Letters*, 2005, 95(25): 253901.

[29] FLOSSMANN F, SCHWARZ U T, MAIER M, *et al*.. Stokes parameters in the unfolding of an optical vortex through a birefringent crystal[J]. *Optics Express*, 2006, 14(23): 11402-11411.

[30] DMITRIEV V G, GURZADYAN G G, NIKOGOSYAN D N. *Handbook of Nonlinear Optical Crystals*[M]. 3rd ed. Berlin: Springer, 1999.

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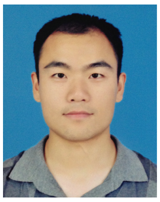

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