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Imaging with a metamaterial "Im-Perfect" lens

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Abstract: There is widespread and strong interest in trying to fabricate a metamaterial in which both the permittivity and permeability are equal to −1 in order to achieve sub-wavelength imaging. Several metamaterial constructs have been proposed with varying degrees of success because of inherent losses, limited bandwidth and scattering from the abstracted circuit elements constituting the artificial material itself. A further limitation is the need to capture evanescent components from the object to be imaged that requires the lens to be located near the object. We have studied the underlying models and constraints that influence the design of a negative index lens and present this analysis as well as reviewing the opportunities. There are inevitable and well-known trade-offs between lens thickness, wavelength, dispersion and absorption. However, these can be characterized both numerically and experimentally, suggesting that a computational imaging approach to the recovery of sub-wavelength features might be effective. Depending on the specific details of the metamaterial employed for imaging, one can consider the data acquired to represent a set of coded apertures.

Key words: metamaterial, negative index, evanescent wave, sub-wavelength imaging

1　Introduction

Pendry's seminal paper on a perfect lens[1], lead to some controversies[2-5], modifications[6] and experimental verifications notably by Liu, *et al.*[7]. It established the surprising result that a slab of negative index material (NIM) amplifies evanescent waves, sustaining them through an NIM slab. This therefore enables perfect imaging in the ideal case and sub-wavelength imaging when the NIM is modestly lossy. Since these evanescent waves carry high spatial frequency information about an object, this theory stimulates the idea of realizing a higher resolution lens, only perfect in the ideal case, but which could find a multitude of applications. Pendry's theory specifically identifies $n = -1$ as the ideal case, resulting in no loss and no reflections. Without a rather thick slab of such a material, the dimensions of the object relative to image plane confine one to the near field, somewhat limiting its advantages over competing methods for high resolution imaging in practice. Also, it takes time for the evanescent components to contribute to the image and the temporal response of such a slab can lead to cloaking and field enhancement anomalies[8].

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The literature on negative index materials (theory and practice) is now extensive with several books written on the subject[9,10]. Despite the controversies and few experimental results demonstrating super-resolution, the field maintains high interest and much promise. The key requirement of course is the successful realization of a material with a bulk effective refractive index of $n = -1$. More precisely, it is well documented[10] that the necessary condition for a negative index is that $\varepsilon' |\mu| + \mu' |\varepsilon| < 0$ for a doubly negative material, while $n = -1$ is usually simply taken to indicate both $\varepsilon' < 0$ and $\mu' < 0$, simultaneously where $\varepsilon = \varepsilon i \varepsilon$. Doubly negative is necessary for transport of evanescent waves and sub-wavelength imaging which is our goal here. Such a material will be a metamaterial of some kind and likely in the short term to be lossy and anisotropic, diminishing the quality of the image. Nevertheless, if information contributes to the image from evanescent waves associated with the object, one can conceive of several techniques to recover a super-resolved image containing meaningful sub-wavelength information, through computational means. One's ability to do this will depend on the precise nature of the metamaterial being used and its scattering characteristics.

When the real parts of ε and μ are not simultaneously negative, the real and imaginary parts of the component of the $\boldsymbol{k}$ vector in the propagation direction, k_z, are non-zero and, as a result, evanescent waves will be transformed inside the NIM to decaying propagating waves. Such transformed evanescent waves with varying spatial frequency k_x will have different values of k_z and, accordingly, be refracted into decaying propagating waves having different directions inside the NIM. Given the fixed thickness of an NIM slab, this will result in different optical phase accumulations for such waves having different spatial frequencies leading to a confused image. Evanescent waves in air do not acquire optical phase in the z direction, but these transformed components will reach the image plane behind the NIM slab with different phases. In addition, effects of absorption and surface roughness will also limit the quality of the image, although one can argue that a small degree of loss acts as a natural regularization parameter for this optical super-resolution procedure[11].

As a result, the NIM slab does not refocus these evanescent waves to a perfect image and the information contained in the evanescent waves is most likely lost. However, if the NIM is truly lossless or exhibits increased transparency through parametric amplification[12] then we could contrive to make $\mathrm{Im}(k_z') \leqslant 0$.

2 Metamaterial constructs and practical constraints

There are many more metamaterial designs and models developed and evaluated at microwave frequencies prior to confronting the challenges of size reduction and the stricter tolerances required for use at optical wavelengths. One possible structure with which we have some experiences is the unit cell proposed by Kong's group[13] and illustrated in Fig. 1. This "S" structure is flexible and has resonances capable of providing negative μ and ε over a common bandwidth.

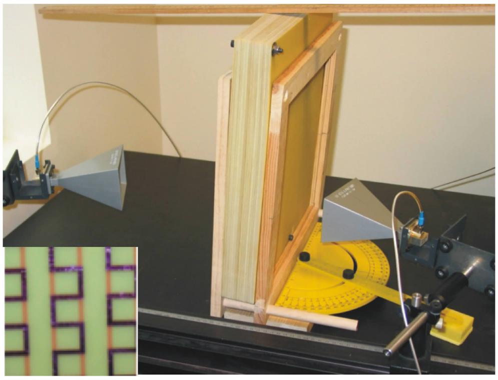

Fig. 1 Typical imaging test arrangement at X-band; insert is example of metamaterial.

We have investigated homogenization and/or effective medium models and recognize that there is no simple procedure to follow to make an array of "S" structures of this kind that will approximate effective bulk negative index properties. A parameterized model for a single unit cell allows line widths, line thicknesses and transverse dimensions to be varied to tune the resonant frequency. A systematic adjustment can be made to adjust both the effective permittivity and permeability to be negative over a common bandwidth but the real challenge lies in doing so at a convenient operating frequency. Data is shown below in Fig. 2 of transmission phase spectra for both TE and TM waves. The height of each "S" was 5 mm and width 2.6 mm and the operating frequency 8.65 GHz. Phase inflections at 8.65 GHz and 9.05 GHz are indicative of a reduced phase velocity and this provides a means to tune the magnetic and electric responses to the same frequency range that can result in a negative phase velocity and effective negative index. One can expect that there will be residual scattering from individual elements in such a metamaterial and the strong periodicity and large metal content, even for subwavelength features, leads to significant scattering, as well as surface wave and diffraction effects, as detailed by Munk[14]. Coherent scattering arising from periodicity can swamp one's interpretation of the emerging image bearing field and losses combined with this strongly angle-dependent scattering allow only poor quality imaging which in this case showed no direct evidence of sub-wavelength resolution. Indeed, it is difficult to estimate the beam displacement that occurs in order to calculate an effective index even crudely, using Snell's law (see Fig. 3). The question to ask is what can be done to improve the situation?

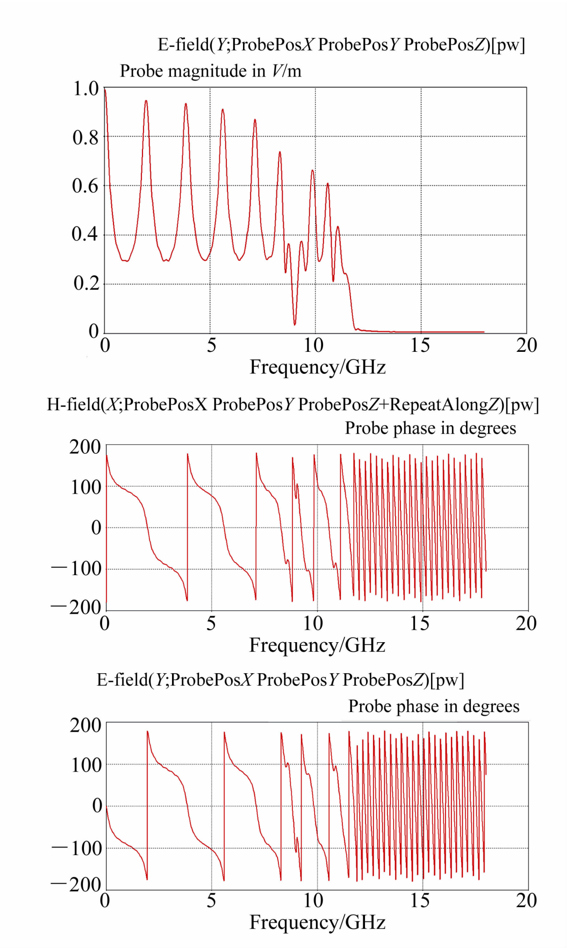

Fig. 2　Top graph shows the transmission spectrum for the metamaterial used and its bandedge at 12 GHz. The lower graphs show the phase spectra for TM and TE propagation, respectively, illustrating phase inflections.

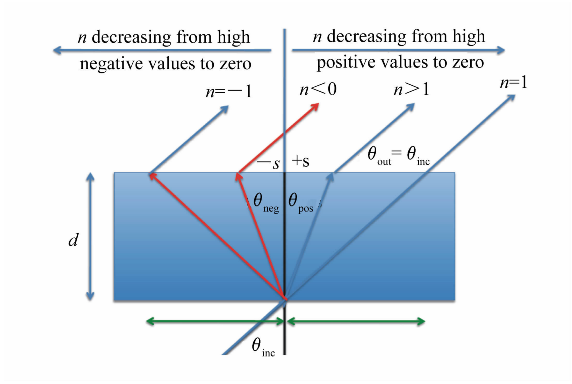

Fig. 3　Pattern of beam deflections (exit displacements ±s) expected as a function of the material's effective bulk refractive index.

To reduce scattering and diffraction effects one needs to make each resonator as small as possible with respect to the operating wavelength, while retaining the resonant frequency with behaviour of the kind we want. No simple inverse method exists that allows one to specify the parameter set for a single "S", quite apart from a three dimensional array of these elements, and obtain the desired bulk index over a chosen bandwidth. It is well established that while resonant behaviour is readily predicted and numerically confirmed for a single "S", there are too many parameters influencing the resonant frequency to easily use this as a design tool for controlling the location and depth of resonance of a distribution. This is disappointing but not critical to short term success since direct(*i. e.* forward solver) simulations can be used, however tedious. A natural limitation of such simulations, assuming the same metamaterial parameters as the physical structure, is to account for edge effects and finite volume resonances. A faster and simpler simulation typically assumes an infinite transverse cross-section.

For the structure shown in Fig. 1, simulations did reveal any unexpected frequency regions in which a negative phase velocity occurred inside the metallic "waveguides" formed by the open regions of each "S". These also corresponded to high index values and mimic published data for fishnet structures that provide frequency dependent index changes but only for normal incidence[15]. This phenomenon seems unlikely to be useful for imaging although one or two possibilities have been published. Super-resolution might be possible if evanescent waves can couple into a "cavity's" slow modes[16] and photonic crystals or other periodic structures are capable of demonstrating negative refraction by engineering equifrequency surfaces[17]. It was also recently reported that metamirror structures can behave like a lens[18].

As mentioned above, it is difficult to assemble the open unit cell structures like the "S" into a bulk material for which one could still predict the resonant frequencies and resonant bandwidths and losses. For example, adding individual "S" elements in the propagation direction reduces capacitance and increases the resonant frequency of the stack. A basic rule of thumb is that the resonant frequency for a metallic split ring structure is roughly proportional to the reciprocal of its size that does not bode well for making sub-wavelength elements. There is an effective L and C for which the resonant frequency can be defined by $1/\sqrt{LC}$ with a bandwidth proportional to $\sqrt{L/C}$ at the micro- and macro-scale. Metal is relatively lossy and is a hindrance to the imaging application leading to the widely used figure of merit $|n'|/n''$. Thinner metal(or conducting) structures are better for reducing losses. Thinner structures also deepen magnetic resonances and reduce electrical resonances while increasing the plasma frequency up to a point[19]. Note also that a high permittivity substrate reduces the effective wavelength that in turn compromises the effective medium approximation[20,21]. For the physical negative index lens structure described here, as with many others having a similar construction, Bragg-like scattering is dominating its response that is compounded by Fabry-Perot resonances arising from the finite thickness of the lens. This does not imply that the observed negative refraction cannot find an appropriate application.

A real material as compared with a meta-material is assumed to have a bulk refractive index that can be explained as an averaged quantity. In defining the dielectric constant of a medium the random phase approximation is usually made when describing induced change in electron density[22]. Most metamaterial structures tend to have periodic properties, with elements of disorder primarily arising from fabrication errors rather than any deliberately introduced positional or dimensional perturbations. We know from the literature that periodic scatterers can exhibit strong dispersion (*e. g.* photonic crystal structures) and band gaps and periodic conductors can support surface waves[14]. The spacing in x, y or z between open resonators affects the resonant frequency and shape of the resonance[23] and as the transverse separation reduces, the collective strength of the resonators increases improving the depth of resonance and hence the range of μ and ε but losses will increase. Increasing the density of closed resonators can increase the bandwidth and depth of the resonance but might introduce polarization dependencies[24]. One can superimpose the resonances of closed resonators[25] and there is a growing literature describing taxonomies of open and closed resonators and how they behave when assembled (*i. e.* tiled)[26]. It is significant that coupling effects between elements are now being modelled and discussed in the same terms as one might describe the interaction of real as opposed to artificial atoms, *i. e.* in terms of hybridization and "bonding modes"[27,28].

In summary, for the imaging application of interest here, we need to reduce spurious scattering and losses. By increasing L and/or reducing R and C we can reduce losses but it must be remembered that we need essentially zero loss and so may need to consider gain for a superlens[29]. If an absorptive loss can be made increasingly narrowband, then from Hilbert transform relations, $\mathrm{Im}\{\varepsilon\}$ or $\mathrm{Im}\{\mu\}$ could approximate a delta function giving a $1/\pi\omega$ resonance behaviour for $\mathrm{Re}\{\varepsilon\}$ or $\mathrm{Re}\{\mu\}$ with little loss. Periodicity within the NIM structure appears to be negligible if the local wavelength is at least 30 × the period[30] and some degree of disorder might appear useful. However, this must be done with care since unit cell disorder can eliminate resonances[31].

Once one has a metamaterial comprised of an array of resonators there are two challenges that remain: (i) do we impose some degree of disorder (not in size but in location) to eliminate coherent diffraction effects or do we already suspect that this will suppress the bulk effective index we want; and (ii) how do we properly characterize the bulk index of the material, *i. e.* estimate the effective permittivity and permeability. Several papers have been written on the latter problem[32,33]. A simplistic approach is to measure the phase delay through a block of the material of thickness d and equate

$$\Delta\phi = -n(\omega)\mathrm{d}\omega/c. \tag{1}$$

One can also fit a simple dispersion curve such as $\varepsilon(\omega)$ or $\mu(\omega) \sim 1-\omega_p^2/[\omega^2-\omega_0^2+\mathrm{i}\Gamma\omega]$, and leave out the ε and μ behaviour. Moreover, one could parameterize these terms directly in terms of the dimensions of the unit structure[34] but as stated earlier, this may require considerable approximations to be made for it to be effective with two- and three-dimensional distributions of these units.

3 Conclusions

Depending on the metamaterial's physical characteristics, *e. g.* material properties and the locations and dimensions of resonant structures, diffraction and scattering effects occur that confuse the interpretation of the exit wavefront in an imaging application. Currently, most materials used to assemble a metamaterial are lossy and evanescent wave amplification is unlikely to occur reducing resolution to more traditional limits. Indeed the angular field of view using a periodic NIM can be severely limited for these reasons[35]. It is possible, however, that higher (*i. e.* evanescent) spatial frequencies might get coupled into propagating waves. Evanescent waves in air are most likely refracted into decaying propagating waves inside a lossy negative index medium, with different spatial frequency components having different propagation directions. These are separated both in time and space; hence there will be no focusing of these specific evanescent waves into a single image plane. Nevertheless, all information encoded by evanescent waves may not be lost in the image domain, and some degree of sub-wavelength imaging might possible. Depending on the specific nature of any diffraction related negative refraction that occurs, and understanding the loss mechanisms, one could characterize the specific modulations of the exit waves. One way to interpret this is to provide a pre-processing or coding of the object wave's spatial frequencies, prior to detection. One is reminded of the wealth of literature that exists, describing codes aperture techniques combined with generalized sampling strategies, that allow a high resolution image to be computed from a set of coded low resolution images(*e. g.* acquired at different incident angles)[36]. Computing is getting faster and cheaper while precision fabrication capabilities to produce increasingly accurate extreme sub-wavelength structures is not. Ultimately, true super-resolution, *i. e.* sub-wavelength imaging, will depend on the enhanced transmission of evanescent waves and the coupling effects between two such evanescent modes, associated with the two faces of the NIM. The future may lie in exploiting resonances in dielectric structures to effect magnetic and electronic responses and dispersive behaviour that mimic high and low index values[37].

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