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Transmission line model and field analysis of metamaterial absorber: ideal concentrator of electromagnetic waves

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Abstract: Arising from the proposed Transmission Line (TL) model for ERR and wire structure, a TL model for a metamaterial absorber is proposed. The S-parameters obtained by this TL model demonstrate the same shapes as the simulation. An investigation of the TL model and average absorption power densities shows that the metamaterial absorber does not simply convert the electromagnetic wave into thermal energy, but concentrate the electromagnetic wave into a small space where it is finally absorbed. This suggests that the metamaterial absorber can be applied to solar cells for the purpose of light trapping.

Key words: metamaterial absorber; Transmission Line model; solar cell

1　Introduction

A Metamaterial Absorber (MA) is a kind of three-layer metamaterial with a thickness significantly smaller than the wavelength, which can absorb an electromagnetic wave completely over a narrow frequency band[1-4]. This unique property makes it an ideal candidate for bolometric pixel elements. However, the mechanism of the unit absorption is still being studied. N. I. Landy has suggested using the matching between the effective permittivity and permeability to interpret this phenomenon[5]. But the strong non-reciprocity of the MA indicates that this interpretation may be inappropriate, because the MA thickness is too small compared to the wavelength. This means that the effective medium theory can not be applied to it and the effective permittivity and permeability approach is not able to describe the MA[6].

Earlier studies have proved that the absorption derives from the ERR structures and is enhanced by the coupling between the ERR and wire structures. We have also tried to understand the time-domain working mechanism of the MA, which leads to the non-reciprocity. However, the nature of the coupling is still not understood nor why the parameters of each layer in the MAs, especially the thickness of the dielectric layer, must be carefully chosen. In this paper, basing on the reported TL model for ERR structure and wire structures[7], a TL model for the MA is proposed. This model provides answers to all

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these questions, can be confirmed by the field analysis, and significantly displays the same non-reciprocity as the simulation. Based on the investigation to the TL model and the distribution of average absorption power densities, it is pointed out that the MA does not simply convert the electromagnetic energy into thermal energy, but concentrates the electromagnetic wave into a small space, and then it is absorbed completely. This working mechanism indicates that the MA is actually an ideal light concentrator and has great potential in the application to the solar cells.

2 TL model for metamaterial absorber

Generally, the MA is constructed from three layers: the ERR layer; the isolation layer; and the wire layer. In the TL model of the MA, it is assumed that the TEM wave propagation through free space and the substrate has intrinsic impedances Z_i and Z_o, respectively, and there is no coupling capacitor or coupling inductor between the ERR layer and wire layer. These two layers are individually modelled as shown in Fig. 1. The TL model of the ERR proposed by Abul K. Azad is used in the ERR layer part of the MA, because it is simple and able to described the behaviour of the S-parameters of the ERR structure. In this TL model, the *LC* resonance and dipole resonance are represented by one set of values of *L*, *C* and *R*, respectively, and the transformer coupling parameter *M* is used to specify the coupling between the two resonances. The wire layer part is modelled by the TL model reported by L. Fu, and the single resonance of the wire structure is expressed by second set of values of *L*, *C* and *R*. The effect of isolation layer in the MA is modelled by a transmission line which connects the ERR and wire structure. All of the parameters of the components need to be optimized so that the S-parameters calculated by the TL model fit the simulation results.

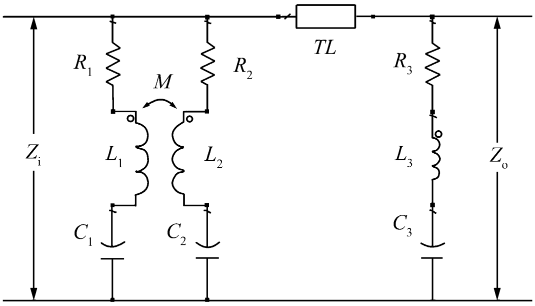


Fig. 1 Transmission line model for the metamaterial absorber. In the ERR structure part, the parameters R_1, L_1, C_1 and R_2, L_2 and C_2 describe the *LC* and dipole resonance, respectively, parameters *M* describe the coupling between the two resonances. Components R_3, L_3 and C_3 specify the resonance of the wire structure and TL refers to the transmission line and represents the isolation layer of the MA.

Once all of the parameters in Fig. 1 are determined, the S-parameter values of the MA can be derived as followed.

The ***ABCD*** matrix of the ERR structure layer, isolation layer and wire structure are, respectively

$$\begin{bmatrix} A_{\mathrm{ERR}} & B_{\mathrm{ERR}} \\ C_{\mathrm{ERR}} & D_{\mathrm{ERR}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{\dfrac{X_1X_2}{X_1+X_2}+M} & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_{\mathrm{iso}} & B_{\mathrm{iso}} \\ C_{\mathrm{iso}} & D_{\mathrm{iso}} \end{bmatrix} = \begin{bmatrix} \cos(kl) & \mathrm{j}Z_{\mathrm{c}}\sin(kl) \\ \dfrac{k\sin(kl)}{Z_{\mathrm{c}}} & \cos(kl) \end{bmatrix}, \quad (1)$$

$$\begin{bmatrix} A_{\mathrm{wires}} & B_{\mathrm{wires}} \\ C_{\mathrm{wires}} & D_{\mathrm{wires}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{X_3} & 1 \end{bmatrix}$$

where $X_1 = \frac{1}{\mathrm{j}\omega C_1} + R_1 + \mathrm{j}w(L_1 - M)$, $X_2 = \frac{1}{\mathrm{j}\omega C_2} + R_2 + \mathrm{j}w(L_2 - M)$, $X_3 = \frac{1}{\mathrm{j}\omega C_3} + R_3 + \mathrm{j}wL_3$, k is the wave vector of the TEM wave, l and Z_c are the thickness and characteristic impedance of the isolation layer, respectively.

Therefore the ***ABCD*** matrix of the MA is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_{\mathrm{ERR}} & B_{\mathrm{ERR}} \\ C_{\mathrm{ERR}} & D_{\mathrm{ERR}} \end{bmatrix} = \begin{bmatrix} A_{\mathrm{iso}} & B_{\mathrm{iso}} \\ C_{\mathrm{iso}} & D_{\mathrm{iso}} \end{bmatrix} = \begin{bmatrix} A_{\mathrm{wires}} & B_{\mathrm{wires}} \\ C_{\mathrm{wires}} & D_{\mathrm{wires}} \end{bmatrix} =$$

$$\begin{bmatrix} \cos(kl) + \dfrac{\mathrm{j}Z_{\mathrm{c}}\sin(kl)}{X_3} & \mathrm{j}Z_{\mathrm{c}}\sin(kl) \\ \left(\dfrac{1}{\dfrac{X_1X_2}{X_1+X_2}+M} + \dfrac{1}{X_3}\right)\cos(kl) + \dfrac{\mathrm{j}\sin(kl)}{X_3Z_{\mathrm{c}}}\left(X_3 + \dfrac{Z_{\mathrm{c}}^2}{\dfrac{X_1X_2}{X_1+X_2}+M}\right) & \cos(kl) + \dfrac{\mathrm{j}Z_{\mathrm{c}}\sin(kl)}{\dfrac{X_1X_2}{X_1+X_2}+M} \end{bmatrix}, \tag{2}$$

and the ***S*** matrix can be calculated as

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{AZ_{\mathrm{o}} + B - (CZ_{\mathrm{o}} + D)Z_{\mathrm{i}}}{AZ_{\mathrm{o}} + B + (CZ_{\mathrm{o}} + D)Z_{\mathrm{i}}} & \dfrac{2\sqrt{Z_{\mathrm{i}}Z_{\mathrm{o}}}}{AZ_{\mathrm{o}} + B + (CZ_{\mathrm{o}} + D)Z_{\mathrm{i}}} \\ \dfrac{2\sqrt{Z_{\mathrm{i}}Z_{\mathrm{o}}}}{AZ_{\mathrm{o}} + B + (CZ_{\mathrm{o}} + D)Z_{\mathrm{i}}} & \dfrac{-AZ_{\mathrm{o}} + B - (CZ_{\mathrm{o}} - D)Z_{\mathrm{i}}}{AZ_{\mathrm{o}} + B + (CZ_{\mathrm{o}} + D)Z_{\mathrm{i}}} \end{bmatrix}, \tag{3}$$

3 Simulation of metamaterials

The first proposed MA was composed of ERR and wire structures, then later MAs with other structures were reported to display the near-unit absorption. Here we take a most familiar and common MA as an example to investigate the validity of the TL model based description of the MA.

The structure of each metal layer in the MA simulated by CST is shown in Fig. 2 with units of μm. The metal in the simulation is gold with conductance of 4.09 × 10^7 S/m and thickness of 800 nm. The space between the ERR structure layer and wires structure layer has thickness 7.8 μm and is filled with polyimide with $\varepsilon = 3.5 + 0.0105\mathrm{i}$, $\mu = 1$. The material of the slice of the MA_1 is GaAs with $\varepsilon = 12.9 + 0.0774\mathrm{i}$, $\mu = 1$. From the positive direction to the negative direction, the system is constructed along the z axis as port 1-vacuum-ERR structure- polyimide-wires structure-GaAs slice-port 2.

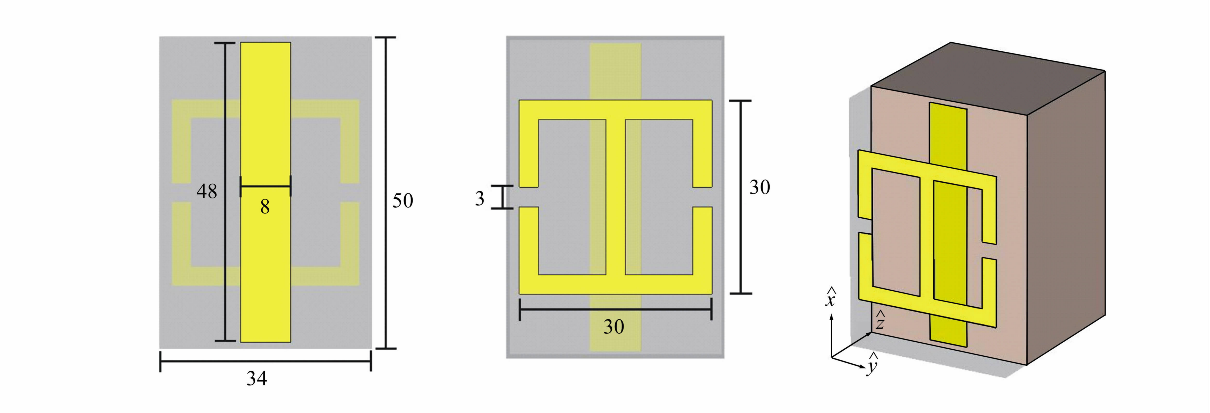
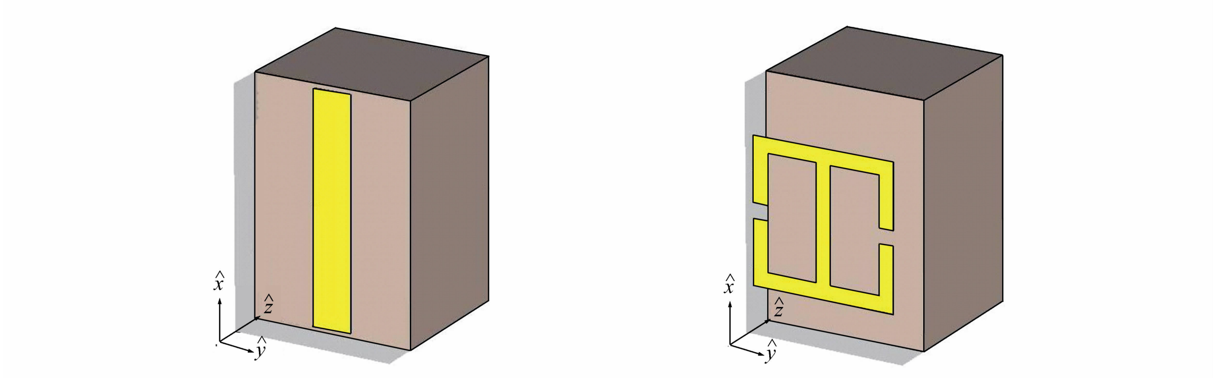


Fig. 2　Dimension details of the MA_1.

In order to determine the L_i, C_i and R_i ($i = 1, 2, 3$) in the TL model, each layer in the MA is simulated. The metamaterial with the same ERR structure (MERR) and the same wire structures (MW) as MA_1 is illustrated in Fig. 3. From the positive direction to the negative direction along the z axis, the

MERR is constructed as port 1-vacuum-ERR structure-polyimide-GaAs slice-port 2, and the MW is constructed as port 1-vacuum-polyimide-GaAs slice-port 2. In all of the simulations, the TEM waves are radiated by port 1 or port 2 and the electric fields are parallel to the x axis.

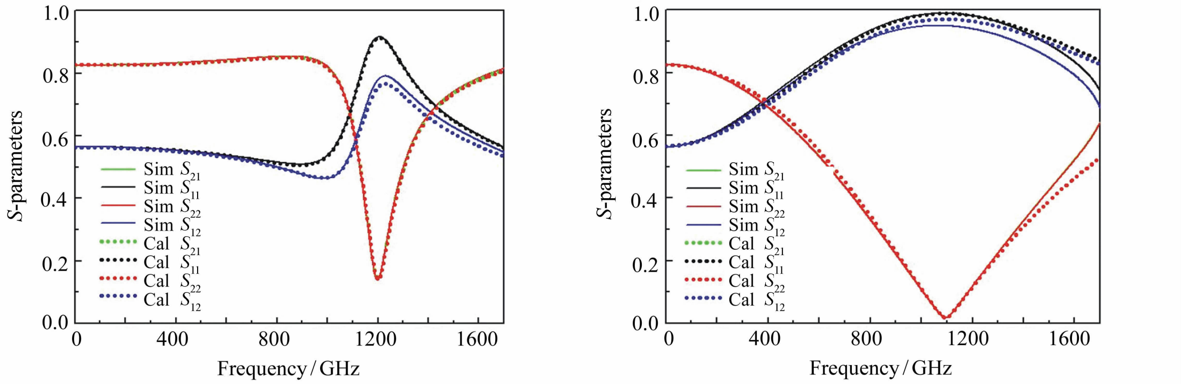


(a) (b)

Fig. 3 Schematic drawing of the MERR(a) and MW(b).

4 Discussion of results

Fig. 4(a) and (b) qunatify the S-parameters of the MERR and MW simulated by the CST and calculated using the TL model in the frequency band 0 − 1 700 GHz. It can be seen that the TL model of the MERR with optimized parameters shows the same S-parameters as the corresponding simulation results across the whole studied frequency region. However, it seems that one group of RLC values is not able to fully describe the MW, because there is a small disagreement between the simulation results and the TL model calculation results of the MW. They share the same S-parameters in the frequency close to and below the resonance point. But the transmittance (S_{21} and S_{12}) of the simulation results rise faster in the higher frequency region up to 0.64 at 1 700 GHz. By contrast, the transmittance calculation results only yield the value 0.55 at 1 700 GHz. There are also some small differences in S_{11} and S_{22} between the simulation and calculation results. Since the discussion is largely focussed on the frequency region near the resonance point, it is reasonable to assume that the TL model can characterize the responses of the MW accurately enough.



(a) (b)

Fig. 4 Simulated and calculated S-parameters of MERR(b) and MW(a).

The simulation results of MA_1 shown in Fig. 5 demonstrate the same behaviour as the experiments do. Utilizing the same optimized parameters in the TL model of both MERR and MW, the TL model calculation results of MA_1 were obtained and shown by the dotted curve in Fig. 5. The calculated results show a strong absorption peak at 1 100 GHz, where most of the input power is absorbed by the component R_1, which is used to represent the loss of LC resonance of the ERR. The S-parameters derived by different methods almost show the same shapes in the frequency band 0 – 1 150 GHz, where S_{12} and S_{21} decrease until the frequency point reaches 1 100 GHz, S_{11} locates at 1 100 GHz with value of 0.15, and S_{22} keeps rising in the low frequency band and yields the maximum about 0.95 at 1 100 GHz. But the variance between the these results become obvious in the frequency band higher than 1 150 GHz. The calculated reflectance S_{11} and S_{22} end up with value 0.9 in the frequency spectrum, and the simulated reflectance displaying the much higher decreasing speed, yields the value about 0.65 at 1 700 GHz. The transmission S_{21} and S_{12}, naturally, shows the opposite variation tendency comparing to the reflectance.

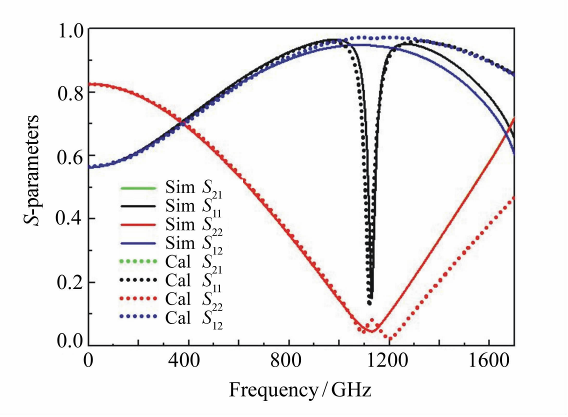


Fig. 5　Simulated and calculated S-parameters of MA_1.

Therefore it can be deduced that the TL model is able to describe the electromagnetic property exactly at frequencies below the absorption peak, but is less accurate in the higher frequency band. As mentioned before, the components in the TL model of MA1 are copied from the TL model of the MERR and MW. When these two structures are put together, the distance between is so small that the coupling capacitance can not be ignored at high frequency. This is thought to be the reason for the discrepancy between the calculated and simulated results.

5　Conclusions

The TL model can describe the MA_1 exactly at frequencies below the absorption peak, but the error becomes larger at higher frequencies. The reason why the thickness isolation layer affects the absorption so much is that the isolation layer actually acts as an impedance transformer in the MA. Both the TL model and the field distribution show that the electromagnetic wave is concentrated in the space near the outside framework of ERR structure and finally absorbed by the isolation layer.

Reference:

[1]　TAO H, LANDY N I, BINGHAM C M, *et al.*. A metamaterial absorber for the terahertz regime: design, fabrication and characterization[J]. *Opt. Express*, 2008, 16(10): 7181-7188.

[2]　TAO H, BINGHAM C M, STRIKWERDA A C, *et al.*. Highly-flexible wide angle of incidence terahertz metamaterial: design, fabrication, and characterization[J]. *Phys. Rev. B*, 2008, 78(24): 241103R.

[3]　AVITZOUR Y, URZHUMOV Y A, SHVETS G. Wide-angle infrared absorber based on a negative-index plasmonic metamaterial[J]. *Phys. Rev. B*, 2009, 79(4): 045131.

[4]　LDNAY N I, BINGHAM C M, TYLER T, *et al.*. Design, theory, and measurement of a polarization insensitive absorber for terahertz imaging[J]. *Phys. Rev. B*, 2009, 79(12): 125104 .

[5] LDNAY N I,SAJUYIGBE S,MOCK J J,*et al.*. Perfect metamaterial absorber[J]. *Phys. Rev. Lett.*,2008,100(20):207402.

[6] XI Y X,XIE Y S,ZHANG H W,*et al.*. The strong non-reciprocity of metamaterial absorber:characteristic, interpretation and modeling[J]. *J. Phys. D:Appl. Phys.*,2009,42(9):095408.

[7] AZAD Z K,TAYLOR A J,SMIRNOVA E,*et al.*. Characterization and analysis of terahertz metamaterials based on rectangular split-ring resonators[J]. *Appl. Phys. Lett.*,2008,92(1):011119.

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